Skeletonization of Three-Dimensional Object Using Generalized Potential Field

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Abstract—The medial axis transform (MAT) is a skeletal representation of an object which has been shown to be useful in interrogation, animation, finite element mesh generation, path planning, and feature recognition. In this paper, the potential-based skeletonization approach for 2D MAT [1], which identifies object skeleton as potential valleys using a Newtonian potential model in place of the distance function, is generalized to three dimensions. The generalized potential functions given in [2], which decay faster with distance than the Newtonian potential, is used for the 3D case. The efficiency of the proposed approach results from the fact that these functions and their gradients can be obtained in closed forms for polyhedral surfaces. According to the simulation results, the skeletons obtained with the proposed approach are closely related to the corresponding MAT skeletons. While the medial axis (surface) is 2D in general for a 3D object, the potential valleys, being one-dimensional, form a more realistic skeleton. Other desirable attributes of the algorithm include stability against perturbations of the object boundary, the flexibility to obtain partial skeleton directly, and low time complexity.

Index Terms—3D skeletonization, medial axis transform, potential field, distance function, 3D thinning.

1 Introduction

The skeleton of an object, as defined by Blum [3], is the locus of the centers of all its interior maximal circles (2D) or spheres (3D). Together with the associated radius function, which is the radius of the maximal ball around any given point on the skeleton, we can represent the object with less information than the object itself. Such a technique can be applied in document encoding [4] and shape representation (or description) [5], [6]. Some other applications of the skeleton can be found in robot path planning [7], feature recognition [8], automatic mesh generation [9], [10], and finite element modeling [11].

Commonly, such representations can be derived first by computing a distance transform which yields the shortest distance from each interior point of a region to its border or, equivalently, by identifying at each point the largest possible size of the primitive of a given shape such that it is entirely contained in a region to be represented. The representation is then derived by identifying the primitives having locally maximal sizes. Such approaches are straightforward, but computationally expensive since the distance computation must be performed at every point of a region.

This paper presents a new concept of obtaining the skeleton of 3D polyhedral regions in a computationally efficient manner. The efficiency of the approach results from the use of an intermediate, analog representation of the given shape information—the potential field. The use of potential field representation helps avoid the expensive task of computing the distance transform at each pixel/voxel and the computation is limited approximately to the locations of the locally maximal primitives. The computation of the potential field itself can be performed efficiently if the shape information is given in a compact form, e.g., the polyhedral representation of the surface. Such a compact description may be directly available as a part of the specification of the given object shape or it may be derived from the given object data such as its surface or volume descriptions.

1.1 The 2D MAT

In the 2D space, the medial axis transform (MAT), or skeleton, of a shape is defined in terms of the medial axis (MA), which is the loci of those points which are equidistant from at least two points on the region border [3]. Thus, the medial axis is composed of the centers of "locally maximal" discs, defined as discs that are as large as they can be without crossing the region border, but are not contained in any other locally maximal discs. The medial axis and the radii of the maximal discs associated with each axis point together define the MAT representation. Fig. 1 shows the MAT skeleton for a rectangular region.

A number of algorithms developed to obtain the skeleton of a digital image region explicitly compute the distance transform for each point inside the region [12], [13], [14], [15], [16], [17]. Then, the definition of the locally maximal discs is used in a straightforward way to identify centers of such discs and, thus, the skeleton. The propagation and extinction of the fire in the grass fire model are directly used to obtain the skeleton in [18]. An algorithm for determining skeletons of polygonal regions based on the same propagation process is presented in [19]. While the computing time is said to be roughly proportional to the number of edges of the polygon, the algorithm is complicated to program. It is shown in [20] that the skeletonization problem is linearly reducible to the construction of generalized Voronoi

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Manuscript received 22 Oct. 1998; revised 2 Aug. 1999; accepted 11 Aug. 2000.
Recommended for acceptance by I. Dinstein.
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